GENERAL APTITUDE

Q. No. 1 - 5 Carry One Mark Each

1. The lecture was attended by quite _______ students, so the hall was not very ________.

   (A) few, quite (B) few, quiet (C) a few, quiet (D) a few, quite

Key: (C)

2. They have come a long way in __________ trust among the users.

   (A) creating (B) create (C) created (D) creations

Key: (A)

3. If E = 10; J = 20; O = 30; and T = 40, what will be P + E + S + T?

   (A) 51 (B) 120 (C) 82 (D) 164

Key: (B)

Sol: Given E = 10, J = 20, O = 30 and T = 40

P + E + S + T ?

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	26

If E = 10 [$\because$ 2 × 5]; J = 20[10 × 2]; O = 30[15 × 2]; T = 40[20 × 2]; then

P = 32[16 × 2]; S = 38[19 × 2]; T = 40[20 × 2]

$\therefore$ P + E + S + T = 32 + 10 + 38 + 40 = 120.

4. The CEO's decision to quit was as shoking to the Board at it was to ________.

   (A) myself (B) I (C) my (D) me

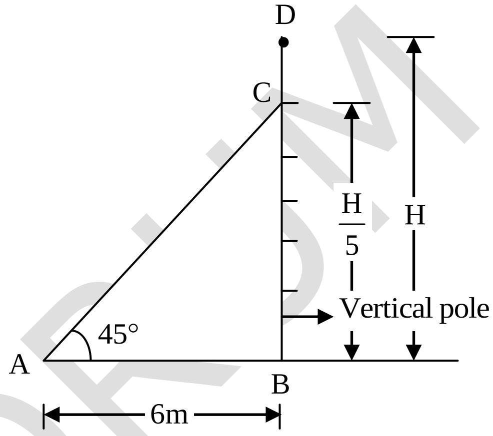
Key: (D)

5. On a horizontal ground, the base of a straight ladder is 6m away from the base of a vertical pole. The ladder makes an angle of 45° to the horizontal. If the ladder is resting at a point located at one-fifth of the height of the pole from the bottom, the height of the pole is ________ meters.

   (A) 35 (B) 30 (C) 15 (D) 25

Key: (B)

Sol: H denotes height of the pole from the bottom.



In $\Delta^{le}ABC$;

$$\tan 45° = \frac{H/5}{6}$$

$$\Rightarrow 1 = \frac{H/5}{6} \qquad [\because \tan 45° = 1]$$

$$\Rightarrow H/5 = 6 \Rightarrow H = 30\,m.$$

Q. No. 6 - 10 Carry Two Marks Each

6. "The increasing interest in tribal characters might be a mere coincidence, but the timing is of interest. None of this, though, is to say that the tribal here has arrived in Hindi cinema, or that the new crop of characters represents the acceptance of the tribal character in the industry. The films and characters are too few to be described as a pattern."

What does the word 'arrived' mean in the paragraph above?

(A) went to a place (B) came to a conclusion

(C) attained a status (D) reached a terminus

Key: (C)

7. In a sports academy of 300 people, 105 play only cricket, 70 play only hockey, 50 play only football, 25 play both cricket and hockey, 15 play both hockey and football and 30 play both cricket and football. The rest of them play all three sports. What is the percentage of people who play at least two sports?

(A) 28.00 (B) 23.30 (C) 50.00 (D) 25.00

Key: (D)

Sol: Given,

Play only cricket $\rightarrow$ 105

Play only hockey $\rightarrow$ 70

Play only football $\rightarrow$ 50

Play both cricket and hockey $\rightarrow$ 25

Play both hockey and football $\rightarrow$ 15

Play both cricket and football $\rightarrow$ 30

Total number of people in sports academy= 300

Number of people who play only one sport = 105+70+50=225

Number of people who play at least two sports=Total-(people who play only one sport) =300-225=75

$\therefore$ Required percentage= $\left[\frac{75}{300}\times 100\right]\% = 25\%$

8. P, Q, R, S and T are related and belong to the same family. P is the brother of S. Q is the wife of P. R and T are the children of the siblings P and S respectively. Which one of the following statements is necessarily FALSE?

(A) S is the aunt of T
(B) S is the aunt of R
(C) S is the sister-in-law of Q
(D) S is the brother of P

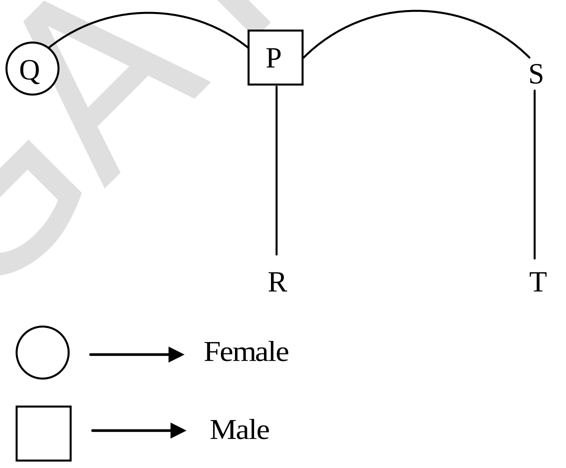
Key: (A)

Sol: Given,

P is the brother of S,

Q is the wife of P,

R and T are the children of the siblings P and S respectively.



$\therefore$ T is son/daughters of S.

$\therefore$ S is the aunt of T is false.

9. The new cotton technology, Bollgard-II, with herbicide tolerant traits has developed into a thriving business in India. However, the commercial use of this technology is not legal in India. Now with standing that, reports indicate that the herbicide tolerant Bt cotton had been purchased by farmers at an average of Rs 200 more than the control price of ordinary cotton, and planted in 15% of the cotton growing area in the 2017 Kharif season.

Which of the following statements can be inferred from the given passage?

(A) Farmers want to access the new technology by paying high price

(B) Farmers want to access the new technology if India benefits from it

(C) Farmers want to access the new technology for experimental purposes

(D) Farmers want to access the new technology even if it is not legal

Key: (D)

10. A square has sides 5 cm smaller than the sides of a second square. The area of the larger square is four times the area of the smaller square. The sided of the larger square is _________ cm.

(A) 18.50 (B) 10.00 (C) 15.10 (D) 8.50

Key: (B)

Sol: Let, Side of smaller square = x cm

Side of larger square = (x + 5) cm

Area of small square = x^2

Area of larger square = $(x + 5)^2$

Given,

$(x+5)^2 = 4x^2$ ($\because$ Area of the large square is four times the area of the smaller square]

$\Rightarrow x^2 + 25 + 10x = 4x^2$

$\Rightarrow 3x^2 - 10x - 25 = 0$

$\Rightarrow x = 5; -\frac{5}{3} \rightarrow$ which is not possible.

$\therefore$ Side if larger square = 10cm

CIVIL ENGINEERING

Q. No. 1 to 25 Carry One Mark Each

1. Consider a two-dimensional flow through isotropic soil along x direction and z direction. If h is the hydraulic head, the Laplace's equation of continuity is expressed as

(A) $\frac{\partial h}{\partial x}+\frac{\partial h}{\partial z}=0$

(B) $\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial x \partial z}+\frac{\partial^2 h}{\partial z^2}=0$

(C) $\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial z^2}=0$

(D) $\frac{\partial h}{\partial x}+\frac{\partial h}{\partial x}\frac{\partial h}{\partial z}+\frac{\partial h}{\partial z}=0$

Key: (C)

Sol: Soil mass is fully saturated.

By applying Darcy's law

$$V_x=K_x.i_x=K_x.\frac{dh}{dx}$$

$$V_z=K_z.i_z-K_z.\frac{dh}{dz}$$

From the continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial z}=0$$

$$K_x.\frac{\partial^2 h}{\partial x^2}+K_z.\frac{\partial^2 h}{\partial z^2}=0$$

For isotropic soil $K_x=K_z$

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial z^2}=0$$

2. The coefficient of average rolling friction of a road is f_r and its grade is +G%. If the grade of this road is doubled, what will be the percentage change in the braking distance (for the design vehicle to come to a stop) measured along the horizontal (assume all other parameters are kept unchanged?

(A) $\frac{0.02G}{f_r+0.01G}\times 100$

(B) $\frac{f_r}{f_r+0.02G}\times 100$

(C) $\frac{2f_r}{f_r+0.01G}\times 100$

(D) $\frac{0.01G}{f_r+0.02G}\times 100$

Key: **(D)**

Sol: **1st Stage**

$\mu_1=f_r,\ n_1=+G\%$

Braking distance (d_1) $=\frac{V^2}{2g(\mu+0.01n)}=\frac{V^2}{2g(f_r+0.01G)}$

2nd Stage

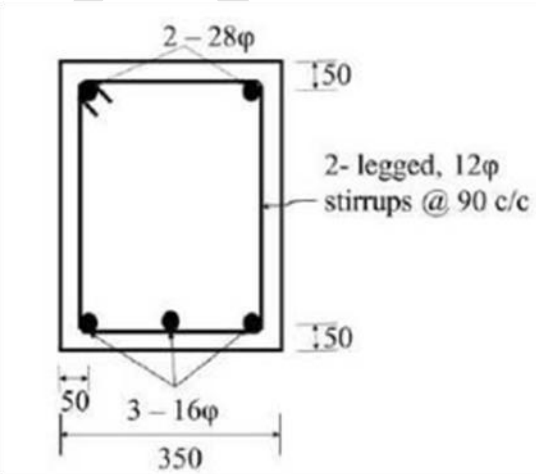
$\mu_2 = f_r,\ n_2 = +2G\%$

Braking distance (d_2) $= \dfrac{V^2}{2g(\mu+0.01n)} = \dfrac{V^2}{2g(f_r+0.01\times 2G)} = \dfrac{V^2}{2g(f_r+0.02G)}$

$$\%\ \text{change} = \frac{d_1-d_2}{d_1}\times 100 = \frac{\dfrac{V^2}{2g(f_r+0.01G)} - \dfrac{V^2}{2g(f_r+0.02G)}}{\dfrac{V^2}{2g(f_r+0.01G)}}\times 100$$

$$\frac{\dfrac{1}{f_r+0.01G} - \dfrac{1}{f_r+0.02G}}{\dfrac{1}{f_r+0.01G}} = \frac{(f_r+0.02G)-(f_r+0.01G)}{f_r+0.02G} = \frac{0.01G}{f_r+0.02G}\times 100$$

3. In the reinforced beam section shown in the figure (not drawn to scale), the nominal cover provided at the bottom of the beam as per IS 456-2000, is



All dimensions are in mm

(A) 36 mm (B) 50 mm (C) 30 mm (D) 42 mm

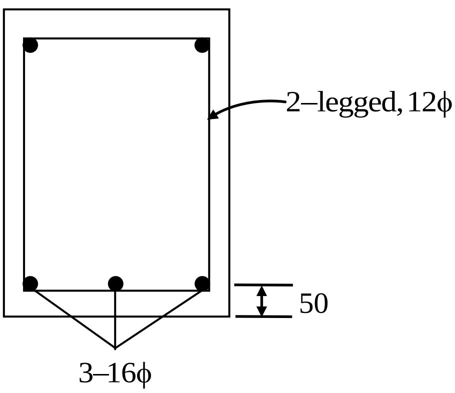
Key: (C)

Sol: Nominal cover

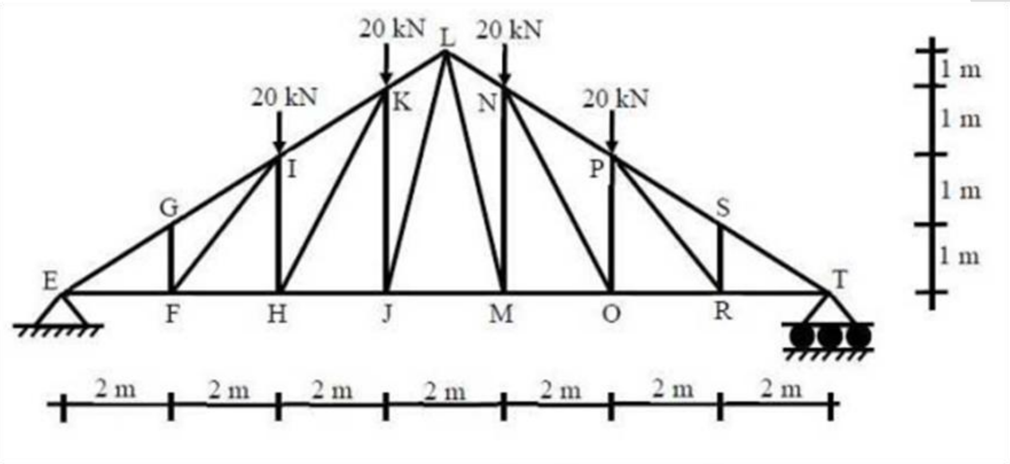
$= \text{Cover} - \dfrac{\text{diameter of bar}}{2} - \text{diameter of stirrup}$

$= 50\text{mm} - \dfrac{16}{2}\text{mm} - 12$

$= 50 - 8 - 12 = 50 - 20 = 30\text{mm}$



4. A plane truss is shown in the figure (not drawn to scale).

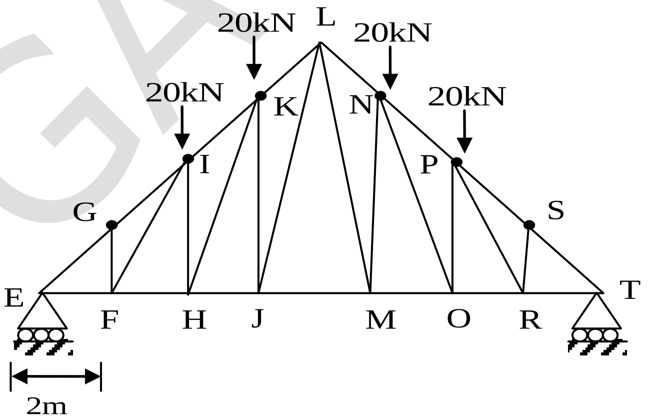


Which one of the options contains ONLY zero force members in the truss?

(A) FG, FI, HI, RS
(B) FI, FG, RS, PR
(C) FI, HI, PR, RS
(D) FG, FH, HI, RS

Key: (B)

Sol: We know that, if three members are meeting at the joint, two of them are collinear and there is no point load acting on the joint, then the third member wil carry zero force.



From the above statement,
We can say the GR, FI, RS, PR members will carry zero force.

5. Which one of the following is a secondary pollutant?

(A) Ozone
(B) Carbon Monoxide
(C) Volatile Organic Carbon (VOC)
(D) Hydrocarbon

Key: (A)

Sol: Based on origin, air pollutants are classified into two categories

(i) Primary air pollutants: Particulates, CO, SO_x, NO_x, hydrocarbons

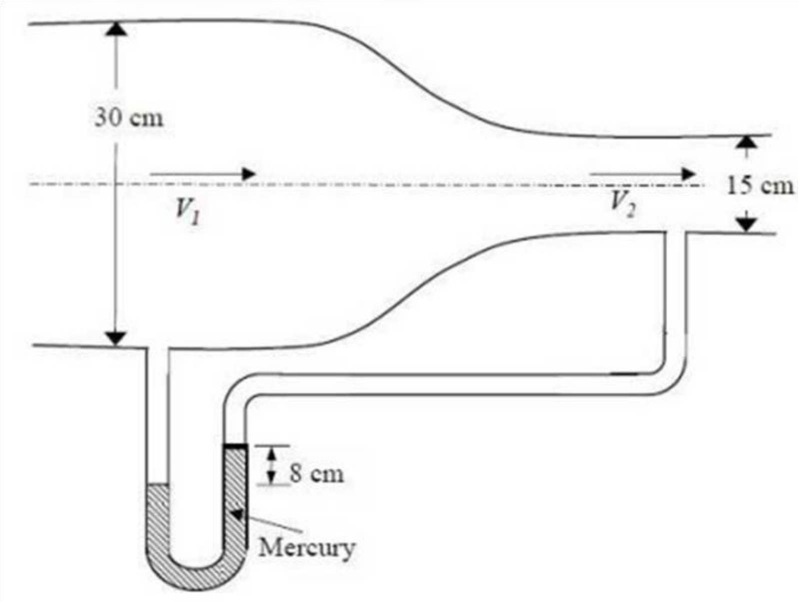
(ii) Secondary air pollutants: Ozone, PAN, PBN, PPN, Smog

6. If the path of an irrigation canal is below the bed level of a natural stream, the type of cross-drainage structure provided is

(A) Aqueduct
(B) Sluice gate
(C) Super passage
(D) Level crossing

Key: (C)

7. A circular duct carrying water gradually contracts from diameter of 30 cm to 15cm. The figure (not drawn to scale) shows the arrangement of differential manometer attached to the duct.

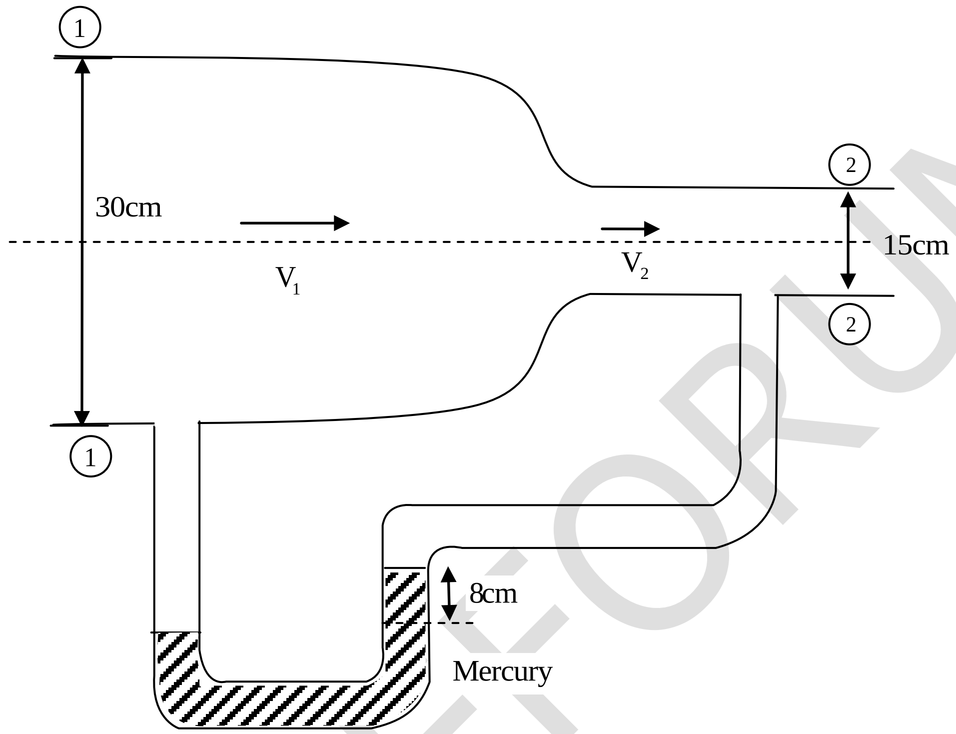


When the water flow, the differential manometer shows a deflection of 8cm of mercury (Hg). The value of specific gravity of mercury and water are 13.6 and 1.0, respectively. Consider the acceleration due to gravity. g = 9.81 m/s^2. Assuming frictionless flow, the flow rate (in m^3/s, rounded off to 3 decimal places) through the duct is _______.

Key: (0.081)

$$\text{Discharge} = Q = C_D \frac{\sqrt{2gH}\ A_1A_2}{\sqrt{A_1^2 - A_2^2}}$$

$$A_1 = \frac{\pi}{4}(0.3)^2;\ A_2 = \frac{\pi}{4}(0.15)^2 \begin{bmatrix} A_1 = A_{rea} \text{ of } C/S(1)-(1) \\ A_2 = \text{Area of } C/S(2)-(2) \end{bmatrix}$$



$$H = \text{Differential head} = \left(\frac{S_{Hg}}{\rho_w} - 1\right) \times \text{sum} = \left(\frac{13.6 \times 1000}{1000} - 1\right) \times 0.08 = 1.008\text{m}$$

$$Q = \frac{\sqrt{2 \times 9.81 \times 1.008} \times \pi/4 \times (0.3)^2 \times \pi/4 \times (0.15)^2}{\sqrt{\left[\pi/4 \times (0.3)\right]^2 - \left[\pi/4 \times (0.15)\right]^2}} \left[C_d \approx 1;\ \text{Frictionless flow}\right]$$

$$= 0.081\,\text{m}^s/\text{s}$$

8. For a given loading on a rectangular plain concrete beam with an overall depth of 500 mm, the compressive strain and tensile developed at the extreme fibers are of the same magnitude of 2.5 × 10^{-4}. The curvature in the beam cross-section (in m^{-1}, round off to 3 decimal places), is _________.

Key: (0.001)

Sol: Overall depth (d) = 500mm

We know that, from bending equation

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

Stress ∝ Strain

$$f \propto \varepsilon \Rightarrow f = E\varepsilon \Rightarrow \frac{f}{E} = \varepsilon$$

$$\frac{f}{y} = \frac{E}{R} \Rightarrow \left(\frac{f}{E}\right) = \frac{y}{R} \Rightarrow 2.5 \times 10^{-4} = y \times \frac{1}{R}$$

$$y = \frac{d}{2} = \frac{500}{2} = 250\text{mm} \Rightarrow \frac{250}{R} = 2.5 \times 10^{-4}$$

Curvature $\left(\frac{1}{R}\right)=\frac{2.5\times10^{-4}}{250}=1\times10^{-6}\,mm^{-1}=1\times10^{-3}m^{-1}=0.001m^{-1}$

9. The maximum number of vehicles observed in any five minute period during the break hour is 160. If the total flow in the peak hour is 1000 vehicles, the five minute peak hour factor (round off to 2 decimal places) is _________.

Key: (0.52)

Sol: Given (maximum number of vehicle) in 5 minutes = 160 vehicles

Flow in the peak hour = 1000 vehicles

$$5\text{ minute-Peak hour factor}=\frac{\text{Peak flow}}{12(5\text{ minute traffic})}=\frac{1000}{12(160)}=0.52$$

10. The interior angles of four triangles are given below:

Triangle	Interior Angles
P	85°, 50°, 45°
Q	100°, 55°, 25°
R	100°, 45°, 35°
S	130°, 30°, 20°

Which of the triangles are ill-conditioned and should be avoided in Triangulation surveys?

(A) Both Q and S (B) Both Q and R (C) Both P and R (D) Both P and S

Key: (A)

Sol: If any interior angle of a triangle is more than $120°$ or less than $30°$; then the triangle is called ill-conditioned.

Triangle Q: One angle is $25°<30°$

Q is ill-conditioned

Triangle S: One angle is $130°<120°$

Other angle is $20°<30°$

S is ill-conditioned

Both Q & S

11. A concentrated load of 500 kN is applied on an elastic half space. The ratio of the increase in vertical normal stress at depths of 2m and 4m along the point of the leading, as per Boussinesq's theory, would be __________.

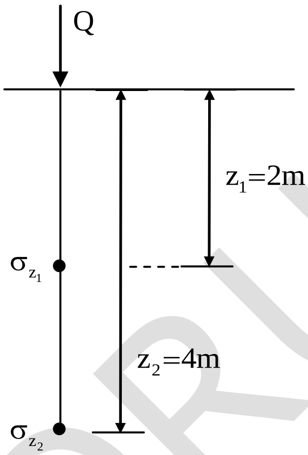
Key: (4)

Sol: As per Boussinegq's theory

Vertical stress, $(\sigma_z) = \frac{Q}{z^2}\frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$

$\sigma_z = K.\frac{Q}{z^2} \Rightarrow \sigma_z \propto \frac{1}{z^2}$

$\frac{\sigma_{z1}}{\sigma_{z2}} = \left(\frac{z_2}{z_1}\right)^2 = \left(\frac{4}{2}\right)^2 = 4$



12. In a soil specimen, the total stress, effective stress, hydraulic gradient and critical hydraulic gradient are σ, σ', i and i_c, respectively. For initiation of quicksand condition, which one of the following statement is TRUE?

(A) $\sigma' = 0$ and $i = i_c$
(B) $\sigma = 0$ and $i = i_c$
(C) $\sigma' \neq 0$ and $i = i_c$
(D) $\sigma' \neq 0$ and $i \neq i_c$

Key: (A)

Sol: In case of upward seepage, force becomes equal to the bouyant weight of the soil. The effective stress in the soil becomes zero $(\sigma = 0)$.

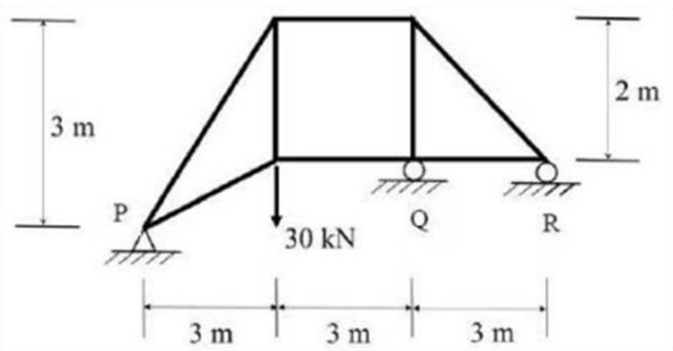
$H.\gamma_{sub} - i.H.\gamma_w = 0$

$H.\gamma_{sub} = iH.\gamma_w$

$i = \frac{\gamma_{sub}}{\gamma_w} = i_{cr} \Rightarrow i = i_{cr}$

For quick sand condition $\sigma' = 0$ and $i = i_{cr}$

13. Consider the pin-jointed plane truss shown in the figure (not drawn to scale). Let RP, RQ, and RR denote the vertical reactions (upward positive) applied by the supports at P, Q, and R, respectively, on the truss. The correct combination of (RP, RQ, and RR) is represented by

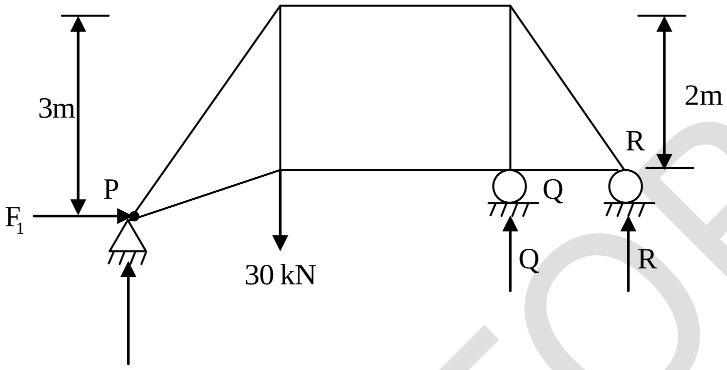


(A) $(30,-30,30)\text{kN}$ (B) $(10,30,-10)\text{kN}$

(C) $(20,0,10)\text{kN}$ (D) $(0,60,-30)\text{kN}$

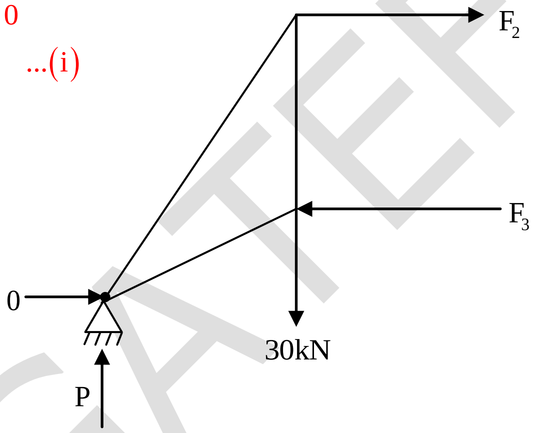
Key: (A)

Sol:



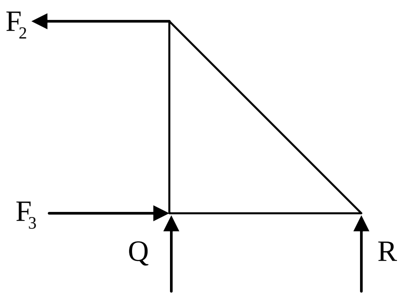
$\Sigma F_H = 0$

$F_1 = 0 \quad ...(i)$



$\Sigma F_v = 0$

$P = 30 \quad ...(ii)$



$\Sigma F_H = 0$

$F_2 = F_2 \quad ...(iii)$

$F_2 \times 3 + 30 \times 3 - F_3 \times 1 = 0$

$F_2 \times 3 - F_2 \times 1 + 90 = 0$

$2F_2 = -90$

$F_2 = -45$

$\Rightarrow F_2 = -45$

$Q + R = 0$

$F_3 \times 2 + R \times 3 = 0$

$-45 \times 2 + R \times 3 = 0$

$R = 30 \Rightarrow Q = -30$

14. A retaining wall of height H with smooth vertical backface supports a backfill inclined at an angle β with the horizontal. The backfill consists of cohesionless soil having angle of internal friction ϕ. If the active lateral thrust acting on the wall is P_a which one of the following statements is TRUE?

(A) P_a acts at a height H/3 from the base of the wall and at an angle β with the horizontal

(B) P_a acts at a height H/3 from the base of the wall and at an angle ϕ with the horizontal

(C) P_a acts at a height H/2 from the base of the wall and at an angle β with the horizontal

(D) P_a acts at a height H/2 from the base of the wall and at an angle ϕ with the horizontal

Key: (A)

15. Assuming that there is no possibility of shear buckling in the web, the maximum reduction permitted by IS 800-2007 in the (low-shear) design bending strength of a semi-compact steel section due to high shear is

(A) 50%
(B) governed by the area of the flange
(C) zero
(D) 25%

Key: (C)

Sol: Case I: Low shear

$V < 0.6V_d$

$M_{d_1} = \frac{Z_e f_y}{\gamma_{m_o}} =$ Design bending strength of a semi-compact section in case of low shear.

Case II: High shear

$V > 0.6V_d$

$M_{d_2} = \frac{Z_e f_y}{\gamma_{m_o}} =$ Design bending strength of a semi-compact section in case of high shear.

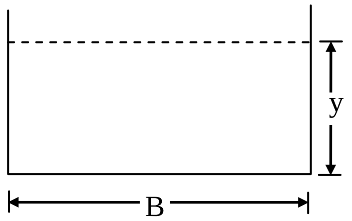
$M_{d_1} = M_{d_2} \Rightarrow$ zero reduction

16. In a rectangular channel, the ratio of the velocity head to the flow depth for critical flow condition, is

(A) $\frac{2}{3}$
(B) 2
(C) $\frac{1}{2}$
(D) $\frac{3}{2}$

Key: (C)

Sol: We know that velocity head = $\frac{V^2}{2g}$



$$\text{Area} = B.y$$

$$\text{Top surface width}\,(T) = B$$

$$Q = A.V$$
$$Q = B.y.V$$

For critical flow condition Froude number $(F_r) = 1 \Rightarrow \frac{Q^2T}{gA^3} = 1$

$$Q^2T = gA^3$$
$$(B.y.V)^2\, T = gA^3$$
$$B^2.y^2.V^2(B) = g(B.y)^3$$
$$B^3.y^2V^2 = g.\,B^3y^3 \Rightarrow \frac{V^2}{g} = y$$

Multiplying with $\frac{1}{2}$ on both sides

$$\frac{V^2}{2g} = \frac{y}{2} \Rightarrow \frac{\frac{V^2}{2g}}{y} = \frac{1}{2} \Rightarrow \frac{\text{Velocity head}}{\text{depth}} = \frac{1}{2}$$

17. A soil has specific gravity of its solids equal to 2.65. The mass density of water is 1000 kg/m^3. Considering zero air voids and 10% moisture content of the soil sample, the dry density (in kg/m^3, round of to 1 decimal place) would be _________.

Key: (2094.9)

Sol: Given that

Specific gravity (G) = 2.65

Mass density of water $(\gamma_w) = 1000\text{kg/m}^3$

Moisture content (w) = 10%

By the zero-air void line

$$\text{dry density}\,(\gamma_{dry}) = \frac{G_s.\gamma_w}{1+wG} = \frac{2.65 \times 1000}{1+0.10\times 2.65} = \frac{2650}{1.265} = 2094.86\ \text{kg/m}^3$$

18. An isolated concrete pavement slab of length L is resting on a frictionless base. The temperature of the top and bottom fibre of the slab are T_t and T_b, respectively. Given: the coefficient of thermal expansion = α and the elastic modulus = E. Assuming $T_t > T_b$ and the unit weight of concrete as zero, the maximum thermal stress is calculated as

(A) $\frac{E\alpha(T_t - T_b)}{2}$ (B) $E\alpha(T_t - T_b)$

(C) zero (D) $L\alpha(T_t - T_b)$

Key: (C)

Sol: As the base of slab is frictionless base $\Rightarrow$ Thermal stress =zero.

19. For a small value of h, the Taylor series expansion for $f(x+h)$ is

(A) $f(x) - hf'(x) + \frac{h^2}{2}f''(x) - \frac{h^3}{3}f'''(x) + ...\infty$

(B) $f(x) - hf'(x) + \frac{h^2}{2!}f''(x) - \frac{h^3}{3!}f'''(x) + ...\infty$

(C) $f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \frac{h^3}{3}f'''(x) + ...\infty$

(D) $f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x) + ...\infty$

Key: (D)

Sol: The Taylor's series expansion for $f(x+h)$ is

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!} + f'''(x) + ...\infty$$

20. Which of one of the following is correct?

(A) $\lim_{x\to 0}\left(\frac{\sin 4x}{\sin 2x}\right) = 1$ and $\lim_{x\to 0}\left(\frac{\tan x}{x}\right) = 1$

(B) $\lim_{x\to 0}\left(\frac{\sin 4x}{\sin 2x}\right) = \infty$ and $\lim_{x\to 0}\left(\frac{\tan x}{x}\right) = 1$

(C) $\lim_{x\to 0}\left(\frac{\sin 4x}{\sin 2x}\right) = 2$ and $\lim_{x\to 0}\left(\frac{\tan x}{x}\right) = \infty$

(D) $\lim_{x\to 0}\left(\frac{\sin 4x}{\sin 2x}\right) = 2$ and $\lim_{x\to 0}\left(\frac{\tan x}{x}\right) = 1$

Key: (D)

Sol: $\lim_{x\to 0}\frac{\sin 4x}{\sin 2x} \quad \left(\frac{0}{0}\right)$

$=\lim_{x\to 0}\frac{4.\cos 4x}{2.\cos 2x}\ [\because \text{from L'hospital rule}]$

$=\frac{4}{2}=2$

$\lim_{x\to 0}\frac{\tan x}{x} \quad \left(\frac{0}{0}\right)$

$=\lim_{x\to 0}\frac{\sec^2 x}{1}=1\ [\because \text{from L'hospital rule}]$

21. An element is subjected to biaxial normal tensile strains of 0.0030 and 0.0020. The normal strain in the plane of maximum shear strain is

(A) 0.0010 (B) Zero (C) 0.0050 (D) 0.0025

Key: **(D)**

Sol: Given $\varepsilon_x = 0.0030;\ \varepsilon_y = 0.0020$

Normal stress in the plane of maximum shear strain is centre of Mohr circle.

Centre of Mohr circle $= \frac{\varepsilon_x+\varepsilon_y}{2} = \frac{0.0030+0.0020}{2} = \frac{0.0050}{2} = 0.0025$

22. The probability that the annual maximum flood discharge will exceed 25000 m^3/s, at least once in next 5 years is found to be 0.25. The return period of this flood event (in years, round off to 1 decimal place) is _____.

Key: **(17.9)**

Sol: Probability of flood discharge exceeds at least once in next 5 years = 0.25

Risk = 0.25

$1-\left(1-\frac{1}{T}\right)^n = 0.25 \Rightarrow 1-0.25 = \left(1-\frac{1}{T}\right)^5$

$(0.75)^{\frac{1}{5}} = 1-\frac{1}{T}$

$\frac{1}{T} = 1-(0.75)^{\frac{1}{5}} \Rightarrow \frac{1}{T} = 1-0.944 = 0.559 \Rightarrow T = 17.89\,\text{years}$

23. A completely mixed dilute suspension of sand particles having diameters 0.25, 0.35, 0.40, 0.45 and 0.50mm are filled in a transparent glass column of diameter 10 cm and height 2.50 m. The suspension is allowed to settle without any disturbance. It is observed that all particles of diameter 0.35 mm settle to the bottom of the column in 30 s. For the same period of 30s, the percentage removal (round off to integer value) of particles of diameters 0.45 and 0.50 mm from the suspension is ________.

Key: (100)

Sol: As we know that settling velocity for discrete particles is given by stokes law as

$$V_T = \frac{(G-1)\gamma d^2}{18\mu} \Rightarrow V_T \propto d^2$$

For 30 second duration if 0.35 mm particle size settles completely then % removal of particle size 0.45 mm and 0.50 mm will be100% respectively for each. As settling velocity of particle size 0.45 mm and 0.50 mm will be greater than settling velocity of size 0.35 mm $\left(V_T \propto d^2\right)$.

24. A simple mass-spring oscillatory system consists of a mass m, suspended from a spring of stiffness k. Considering z as the displacement of the system at any time t, the equation of motion for the free vibration of the system is $m\ddot{z} + kz = 0$. The natural frequency of the system is

(A) $\frac{k}{m}$ (B) $\sqrt{\frac{m}{k}}$ (C) $\sqrt{\frac{k}{m}}$ (D) $\frac{m}{k}$

Key: (C)

Sol: $m.\ddot{z} + kz = 0$

$$\ddot{z} + \frac{k}{m}.z = 0$$

Comparing with $\dot{x} + \omega_n^2 x = 0 \Rightarrow \omega_n^2 = \frac{k}{m}$

Natural frequency, $(\omega_n) = \sqrt{\frac{k}{m}}$

25. A catchment may be idealised as a rectangle There are three rain gauges located inside the catchment at arbitrary locations. The average precipitation over the catchment is estimated by two methods: (i) Arithmetic mean (P_A) and (ii) Thiessen polygon (P_T). Which of the following statements is correct?

(A) P_A is always equal to P_T

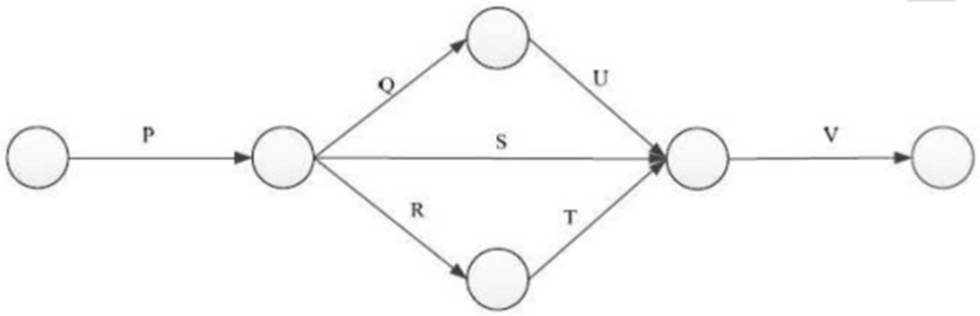
(B) There is no definite relationship between P_A and P_T

(C) P_A is always greater than P_T

(D) P_A is always smaller than P_T

Key: (B)

26. The network of a small construction project awarded to a contractor is shown in the following figure. The normal duration, crash duration, normal cost, and crash cost of all the activities are shown in the table. The indirect cost incurred by the contractor is INR 5000 per day.



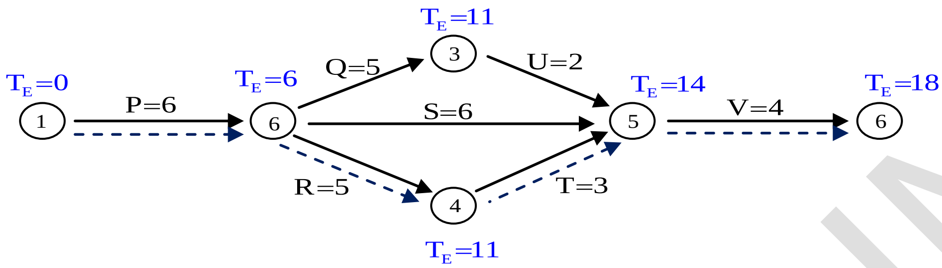
Activity	Normal Duration (days)	Crash Duration (days)	Normal cost (INR)	Crash Cost (INR)
P	6	4	15000	25000
Q	5	2	6000	12000
R	5	3	8000	9500
S	6	3	7000	10000
T	3	2	6000	9000
U	2	1	4000	6000
V	4	2	20000	28000

If the project is targeted for completion in 16 days, the total cost (in INR) to be incurred by the contractor would be ___________.

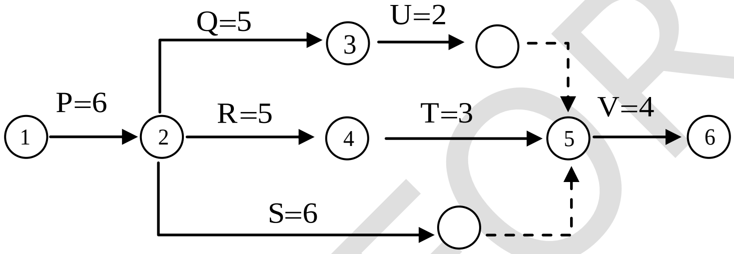
Key: (149500)

Sol:

Activity	Normal duration	Crash duration	Normal cost	Crash cost	C/S slope= $\frac{C.C-N.C}{N.D-C.D}$
P	6	4	15000	25000	5000
Q	5	2	6000	12000	2000
R	5	3	8000	9500	750
S	6	3	7000	10000	1000
T	3	2	6000	9000	3000
U	2	1	4000	6000	2000
V	4	2	20000	28000	4000



Critical path 1 P 2 R 4 T 5 V 6



Indirect cost = Rs. 500

Crashing possibility = 18 –17 = 1 day

To reduce the project duration by 1 day, the following options available.

Crashing ‘R’ by 1 day

After crashing ‘R’ by 1 day, the new network is as for follows.

Path	**Duration**
P-Q-U-V	17
P-S-V	16
P-R-T-V	17

Option	**Cost slope**
P	5000
V	4000
Q & R	2000 +750 = 2750
U & R	2000 + 750 = 2750
Q & T	2000 + 3000 = 5000
U & T	2000 + 3000 = 5000

After crashing Q & R by 1 day, the new network is as follows.

Project duration = 16 days.

Total project cost = Total normal cost + crashing of ‘R’ by 1 day + crashing cost of Q & R by 1 day + indirect cost/day × Project duration

= 15000 + 6000 + 8000 + 7000 + 6000 + 4000 + 20000 + 750 × 1 + 2750 × 1 + 5000 × 16

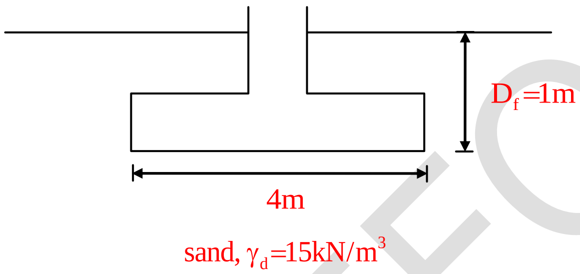
= 1, 49, 500/-

27. A square footing of 4m side is placed at 1 m depth in a sand deposit. The dry unit weight (γ) of sand is 15 kN/m^3. This footing has an ultimate bearing capacity of 600 kPa. Consider the depth factors; $d_q = d_\gamma = 1.0$ and the bearing capacity factor: $N_\gamma = 18.75$. This footing is placed at a depth of 2m in the same soil deposit. For a factor of safety of 3.0 per Terzaghi's theory, the safe bearing capacity (in kPa) of this footing would be ______.

Key: (270)

Sol: Given that soil is sand $\Rightarrow C = 0$

Also given ultimate bearing capacity $(q_u) = 600kPa$



As per Terzaghi's theory, For square footing

Ultimate bearing capacity $(q_u) = 1.3CN_C + \gamma D.N_q + 0.4\gamma B.N_\gamma$

$$600kPa = 0 + 15\times 1\times N_q + 0.4\times 15\times 4\times 18.75$$

$$600 = 15N_q + 450 \Rightarrow N_q = \frac{600-450}{15} = \frac{150}{15} = 10$$

$$\therefore N_q = 10$$

Now, the depth of footing $(D_f) = 2m$

Net ultimate bearing capacity $(q_{nu}) = 1.3\times CN_C + \gamma D(N_q - 1) + 0.4\gamma B\times N_\gamma$

$$= 0 + 15\times 2\times (10-1) + 0.4\times 15\times 4\times 18.75$$

$$= 270 + 450 = 720kPa$$

Safe bearing capacity $(q_{safe}) = \frac{q_{nu}}{F.O.S} + \gamma D_f = \frac{720}{3} + 15\times 2 = 240 + 30 \Rightarrow q_{safe} = 270\,kPa$

28. Which one of the following is NOT a correct statement?

(A) The function $|x|$ has the global minima at x = 0

(B) The function $\sqrt[x]{x}, (x > 0)$, has the global minima at x = e

(C) The function $\sqrt[x]{x}, (x > 0)$, has the global maximum at x= e

(D) The function x^3 has neither global minima nor global maxima

Key: (B)

Sol: (1), (4) are False, Since (x) has global minimum at x=0 and x^3 has neither maximum nor minimum

$$\text{Let } y = x\sqrt{x} = x^{\frac{1}{x}} \Rightarrow \ell ny = \ell nx^{\frac{1}{x}} = \frac{\ell nx}{x} \Rightarrow y = e^{\frac{\ell nx}{x}}$$

$\therefore$ If $\frac{\ell nx}{x}$ is maximum then $y = \frac{\ell nx}{x}$ is maximum for same 'x'

If $\frac{\ell nx}{x}$ is minimum then $y = \frac{\ell nx}{x}$ is minimum for same 'x'

$\therefore$ Consider the function $f(x) = \frac{\ell nx}{x}$

$$\Rightarrow f'(x) = 0 \Rightarrow \frac{x\left(\frac{1}{x}\right) - \ell nx}{x^2} = 0 \Rightarrow \frac{1-\ell nx}{x^2} = 0$$

$\Rightarrow x = e \rightarrow$ stationary point

$$f''(x) = \frac{x^2\left[0 - \frac{1}{x}\right] - [1-\ell nx]2x}{x^4}$$

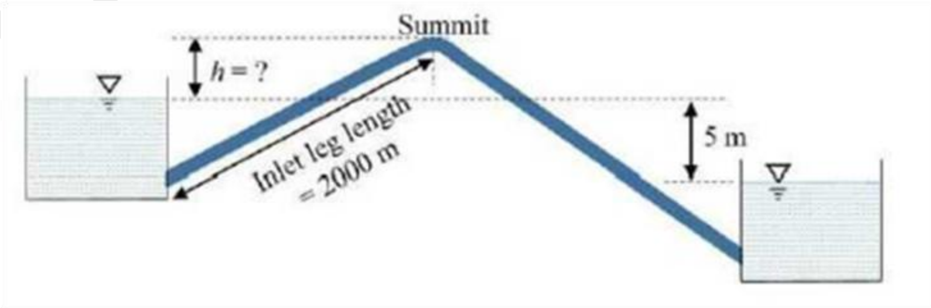
$$\Rightarrow f''(x) = \frac{-x - 2x + 2x\ell nx}{x^4} = \frac{-3x + 2x\ell nx}{x^4}$$

$$\therefore f''(e) = \frac{-3e + 2e(1)}{e^4} = \frac{-e}{e^4} = -e^{-3} < 0$$

$\therefore x = e$ is a point of maximum

$\therefore$ $y = e^{\frac{\ell nx}{x}} \Rightarrow y = \sqrt[x]{x}$ has global maximum at $x=e$.

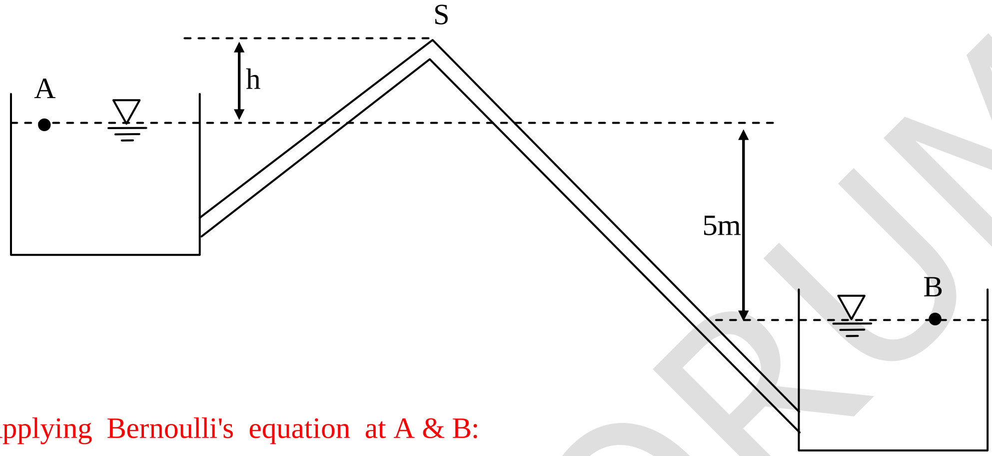
29. Two water reservoirs are connected by a siphon (running full) of total length 5000 m and diameter of 0.10 m, as shown below (figure not drawn to scale).



The inlet leg length of the siphon to its summit is 2000 m. The difference in the water surface levels of the two reservoirs is 5m. Assume the permissible minimum absolute pressure at the summit of siphon to be 2.5m of water when running full. Given: friction factor f = 0.02 throughout, atmospheric pressure = 10.3 m of water, and acceleration due to gravity g = 9.81 m/s^2. Considering only major loss using Darcy Weisbach equation the maximum height of the summit of siphon from the water level of upper reservoir, h (in m round off to 1 decimal place) is ________.

Key: (5.8)

Sol:



Applying Bernoulli's equation at A & B:

$$\frac{P_A}{\gamma_w}+\frac{V_A^2}{2g}+Z_A=\frac{P_B}{\gamma_w}+\frac{V_B^2}{2g}+Z_B+h_t$$

$P_A=P_B=P_{atm};\quad V_A=V_B=0$

$$\Rightarrow Z_A-Z_B=h_f=\frac{f\ell V^2}{2gd}$$

$$\Rightarrow 5=\frac{0.02\times5000\times V^2}{2\times9.81\times0.1}\Rightarrow V=0.313\text{m/s}$$

$$\left[\begin{array}{l} V=\text{Velocity in pipe}\\ \ell=5000\text{m}\\ d=0.1\\ f=0.02\\ h_f=Z_A-Z_B=5\end{array}\right]$$

Applying Bernoulli's equation at A & S:

$$\frac{P_A}{\gamma_w}+\frac{V_A^2}{2g}+Z_A=\frac{P_S}{\gamma_w}+\frac{V_S^2}{2g}+Z_S+h_f$$

$$\left[\begin{array}{l} V_S=V\\ P_A=P_{atm}\\ V_A=0\end{array}\right]$$

$$\frac{P_S}{\gamma_w}=2.5\text{m}$$

$$\frac{P_{Ar}}{\gamma_w}=\frac{P_{atm}}{\gamma_w}=10.326\text{m}$$ [P_{atm} is equivalent to 10.326 m head of water]

$$10.326+Z_A=2.5+\frac{V^2}{2g}+Z_s+h_f$$

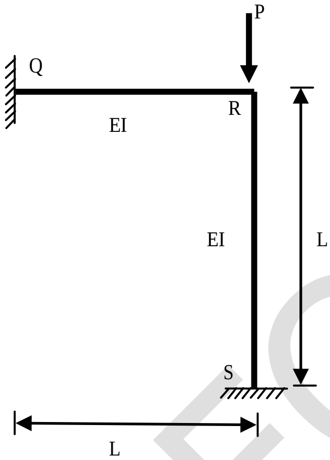
$$\Rightarrow Z_S-Z_A=h;$$

h_f = friction head loss in pipe along inlet leg $=\dfrac{f\ell V^2}{2gd}$ $\left[\begin{array}{l}\ell=2000\text{m}\\ v=0.313\text{m/s}\end{array}\right]$

$$\Rightarrow 10.326-2.5=\frac{(0.313)^2}{2\times9.81}+h+\frac{0.02\times2000\times(0.313)^2}{2\times9.81\times0.1}$$

$$\Rightarrow h=5.823\text{m}\approx5.8\text{m}$$

30. The rigid-jointed plane frame QRS shown in the figure is subjected to a load P at the joint R. Let the axial deformations in the frame be neglected. If the support S undergoes a settlement of $\Delta = \frac{PL^3}{\beta EI}$, the vertical reaction at the support S will become zero when β is equal to



(A) 0.1 (B) 7.5 (C) 3.0 (D) 48.0

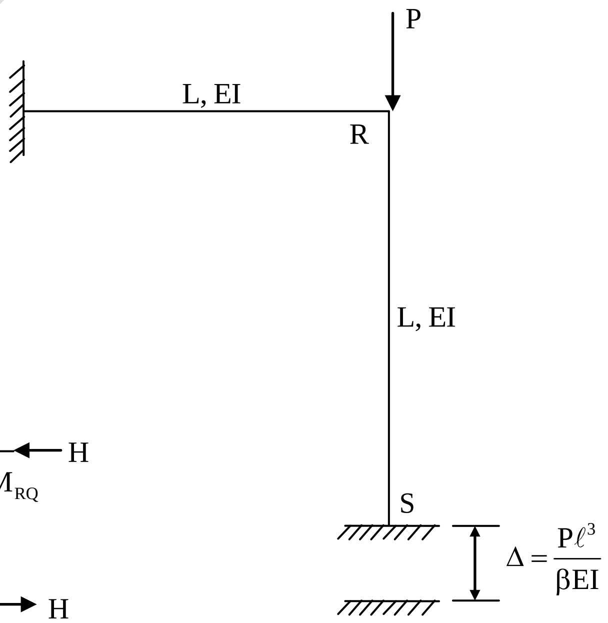
Key: **(B)**

Sol: Using slope deflection method,

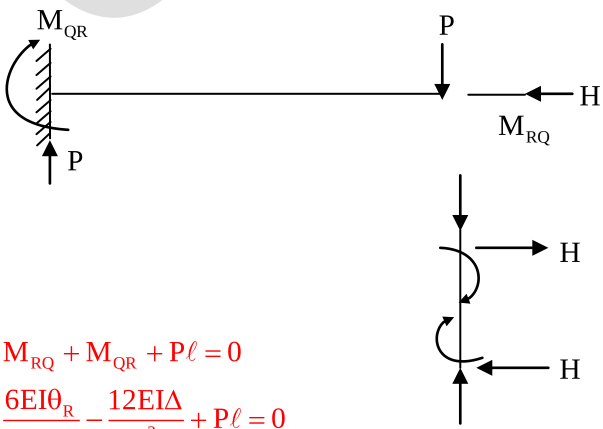
$$M_{QR} = \frac{2EI}{\ell}\left(\theta_R - \frac{3\Delta}{\ell}\right)$$

$$M_{RQ} = \frac{2EI}{\ell}\left(2\theta_R - \frac{3\Delta}{\ell}\right)$$

$$M_{RS} = \frac{2EI}{\ell}\left(2\theta_R\right)$$



If reaction at S is equal to zero



$$M_{RQ} + M_{QR} + P\ell = 0$$

$$\frac{6EI\theta_R}{\ell} - \frac{12EI\Delta}{\ell^2} + P\ell = 0$$

$$\frac{6EI\theta_R}{\ell} - \frac{12EI}{\ell^2} \times \frac{P\ell^3}{\beta EI} + P\ell = 0$$

$$\frac{6EI\theta_R}{\ell} - \frac{12P\ell}{\beta} + P\ell \qquad ....(i)$$

From equilibrium of joint

$$M_{RQ} + M_{RS} = 0$$

$$\frac{8EIQ_R}{\ell} - \frac{6EI\Delta}{\ell^2} = 0$$

$$\frac{6EI\theta_R}{\ell} = \frac{6}{8}\left(\frac{6EI}{\ell^2} \times \frac{P\ell^3}{\beta EI}\right)$$

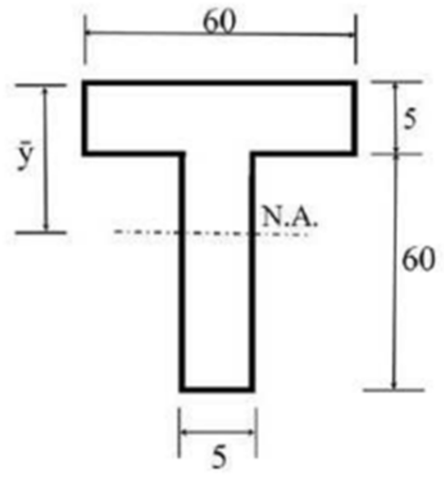
$$\frac{6EI\theta_R}{\ell} = \frac{36\,P\ell}{8\beta} \quad ....(ii)$$

$\Rightarrow$ From (i) & (ii)

$$\frac{36P\ell}{8\beta} - \frac{96P\ell}{8\,\beta}P\ell = 0 \Rightarrow -\frac{60P\ell}{8\beta} + P\ell = 0 \Rightarrow 8\beta = 60$$

$$\beta = \frac{60}{8} = 7.5$$

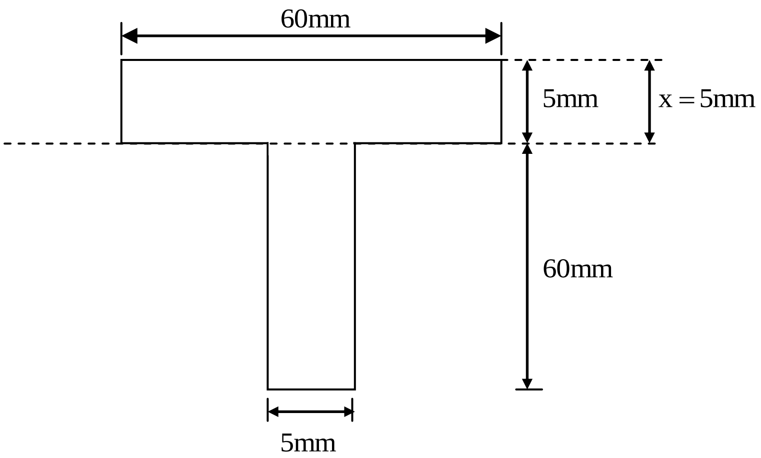
31. If the section shown in the figure turns from fully-elastic to fully-plastic, the depth of neutral axis (N.A), $\bar{y}$, decreases by



(A) 13.75 mm (B) 15.25 mm (C) 10.75 mm (D) 12.25 mm

Key: (A)

Sol:



To locate the equal area axis, equate the area of both side

$$60\times5+(x-5)\times5=(65-x)x5$$
$$300+5x-25=325-5x$$
$$10x=325+25-100=50\Rightarrow x=5mm$$

To calculate the centroidal axis, taking the moment of area about bottom

$$\bar{x}=\frac{60\times5\times30+60\times5\times\left(60+\frac{5}{2}\right)}{60\times5+60\times5}$$
$$=\frac{9000+18750}{600}=46.25\text{ mm from bottom}$$
$$\bar{x}=65-46.25=18.75\text{mm from top}$$

For plastic state

Area in compression = Area in tension

$60\times5=60\times5\Rightarrow$ N.A N.A lies at the intersection web and flangE

$\bar{y}=5$mm from top

Difference in N.A = 18.75-5=13.75 mm

32. Traffic on a highway is moving at a rate 360 vehicles per hour at a location. If the number of vehicles arriving on this highway follows Poisson distribution, the probability (round off to 2 decimal places) that the headway between successive vehicles lies between 6 and 10 seconds is ______

Key: (0.18)

Sol: $\lambda(\text{rate})=360\text{ vehicle/hour}=0.1\text{ vehicle/hour}$

Let 'X' denote the headway between two successive arrivals. [clearly exponential R.V].

$$\therefore f(n)=\lambda e^{-\lambda x}[\text{pdf of } x]\rightarrow \text{exponential distribution}$$
$$\Rightarrow f(n)=0.1e^{-1.0x}$$
$$\therefore \text{Required probability}$$
$$P\left[6<\underset{6<x<10}{\underset{\downarrow}{x}}<10\right]=\int_6^{10}f(n)\,dx$$
$$=\int_6^{10}0.1\,e^{-0.1x}=0.1\left.\frac{\left[e^{-0.1x}\right]}{(-0.1)}\right|_6^{10}=-\left[e^{-0.1(10)}-e^{-0.1(6)}\right]=-\left[e^{-1}-e^{-0.6}\right]=e^{-0.6}-e^{-1}$$
$$\Rightarrow P[6<x<10]=0.18.$$

33. For the following statements:

P – The lateral stress in the soil while being tested in an oedometer is always at-rest.

Q – For a perfectly rigid strip footing at deeper depths in a sand deposit, the vertical normal contact

R – The corrections for overburden pressure and dilatancy are not applied to measured SPT-N values in case of clay deposits.

The correct combination of the statements is

(A) P – TRUE; Q – TRUE; R– TRUE

(B) P – FALSE; Q – FALSE; R – TRUE

(C) P – TRUE; Q– TRUE; R– FALSE

(D) P – FALSE; Q– FALSE; R – FALSE

Key: (C)

34. Average free flow speed and the jam density observed on a road stretch are 60 km/h and 120 vehicles/km, respectively. For a linear speed-density relationship, the maximum flow on the road stretch (in vehicles/h) is ____.

Key: (1800)

Sol: Free flow speed $(V_f) = 60\,\text{km/hr}$

Jam density $(K_J) = 120\,\text{veh/km}$

For a linear speed –density relationship

Maximum flow $(q_{max}) = \dfrac{V_f \times K_J}{4}$

$$= \frac{60 \times 120}{4} (\text{km/hr} \times \text{veh/km})$$

$$q_{max} = 1800 \text{ veh/hr}$$

35. Consider the ordinary differential equations $x^2 \dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} + 2y = 0$. Given the values of y(1) = 0 and y(2) = 2, the value of y(3) (round off to 1 decimal place), is _______.

Key: (6)

Sol: Given D.E

$$x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0 \;\&\; y(1) = 0,\; y(2) = 2.$$

$\rightarrow (1)$ cauchy – Euler linear D.E.

$\Rightarrow \left[x^2D^2 - 2xD + 2\right]y = 0$

Let $xD = \theta;\ x^2D^2 = \theta(\theta-1)$; where $\theta = \dfrac{d}{dz}$ & $x = e^z$

$\left[\theta(\theta-1) - 2\theta + 2\right]y = 0$

$\Rightarrow \left[\theta^2 - 3\theta + 2\right]y = 0$

The A.E is $\theta^2 - 3\theta + 2 = 0$

$\Rightarrow (\theta-2)(\theta-1) = 0$

$\Rightarrow \theta = 1, 2 \rightarrow$ real & distinct

$\therefore$ The solution of eq(1) is

$y = C_1e^z + C_2e^{2z}$

$\Rightarrow y = C_1x + C_2x^2 \left[\because x = e^z\right] \ldots (2)$

Given $\therefore$ $y = 0$ at $x = 1$

$(2) \Rightarrow 0 = C_1 + C_2 \Rightarrow C_1 = -C_2$ & $y = 2$ at $x = 2$

$(2) \Rightarrow 2 = 2C_1 + 4C_2 \Rightarrow 2 = -2C_2 + 4C_2$

$C_2 = 1$ and $C_1 = -1$

From (2); the solution is

$y = -x + x^2 \Rightarrow y(3) = -3 + 3^2 \Rightarrow y(3) = 6.$

36. Sedimentation basin in a water treatment plant is designed for a flow rate of 0.2 m^3/s. The basin is rectangular with a length of 32m, width of 8m, and depth of 4m. Assume that the settling velocity of these particles is governed by the Stokes' law. Given: density of the particles = 2.5 g/cm^3; density of water = 1 g/cm^3; dynamic viscosity of water = 0.01 g/(cm.s); gravitational acceleration =980 cm/s^2. If incoming water contains particles of diameter 25μm (spherical and uniform), the removal efficiency of these particles is

(A) 78% (B) 51% (C) 100% (D) 65%

Key: (D)

Sol: We know that

Settling velocity

$$(V_s) = \frac{g(\rho_s - \rho_\omega)d^2}{18\mu}$$

$$V_s = \frac{980 \times (2.51-)\left(25\times10^{-4}\right)^2}{18\times0.01} = 0.051\,\text{cm/sec}$$

Given flow rate (Q) = 0.2 m^2/sec

We know that surface overflow rate $(V_0) = \dfrac{\text{Volume / time}}{\text{surface area}}$

$$V_0 = \frac{Q}{\text{surface area}}$$

$$\text{surface area} = B \times L = 8 \times 32 = 256\text{m}^2$$

$$V_0 = \frac{Q}{BL} = \frac{0.2\,\text{m}^3/\text{sec}}{256} = 7.8125 \times 10^{-4}\ \text{m/sec}$$

$$V_0 = 7.8125 \times 10^{-2}\ \text{cm/sec}$$

$$\text{Particle removal efficiency } (\eta) = \frac{V_s}{V_0} \times 100$$

$$\eta = \frac{0.051}{0.078} \times 100 = 65.38\%$$

37. A wastewater is to be disinfected with 35mg/L of chlorine to obtain 99% kill of microorganisms. The number of micro-organisms remaining alive (N_t) at time t, is modelled by $N_t = N_0\, e^{-kt}$, where N_0 is number of micro-organisms at t = 0, and k is the rate of kill. The wastewater flow rate is 36 m^3/h, and k = 0.23 min^{-1}. If the depth and width of the chlorination tank are 1.5 m and 1.0m, respectively, the length of the tank (in m, round off to 2 decimal places) is ________.

Key: **(8)**

Sol: Waster water flow rate $(Q) = 36\,\text{m}^3/\text{hr}$

Model is $N_t = N_0 . e^{-Kt}$

$N_t =$ Number of micro-organism survived at time = t = 1% = 0.01

$N_0 =$ Number of micro-organism at t = 0

$$N_t = N_0 . e^{-Kt}$$

$$0.01 = 1.e^{-0.23 \times t}$$

$$e^{-0.23t} = 0.01$$

$$-0.23t = \text{In. } 0.01$$

$$-0.23t = -4.605$$

$$t = 20 \text{ minutes}$$

$$\text{Volume} = Q \times t_d = 36\,\text{m}^3/\text{hr} \times 20 \text{ minutes} = 36\frac{\text{m}^3}{\text{hr}} \times \frac{20}{60}\text{ hrs}$$

$$\text{Volume} = 12\ \text{m}^3$$

$$BLH = 12$$

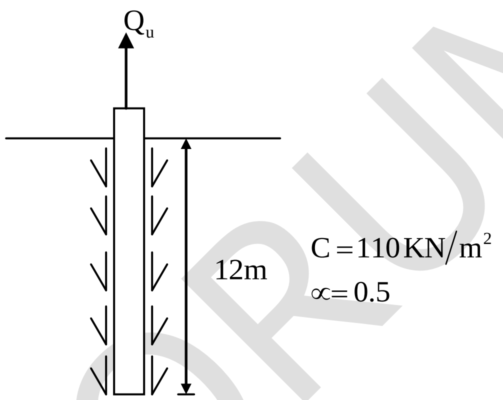
$$1 \times L \times 1.5 = 12$$

$$\text{Length} = \frac{12}{1.5} = 8\text{m}$$

38. A reinforced concrete circular pile of 12m length and 0.6 m diameter is embedded in stiff clay which has an undrained unit cohesion of 110 kN/m^2. The adhesion factor is 0.5. The Net Ultimate Pullout (Uplift) Load for the pile (in kN, round off to 1 decimal place is) is ________.

Key: **(1244.07)**

Sol: For Clays



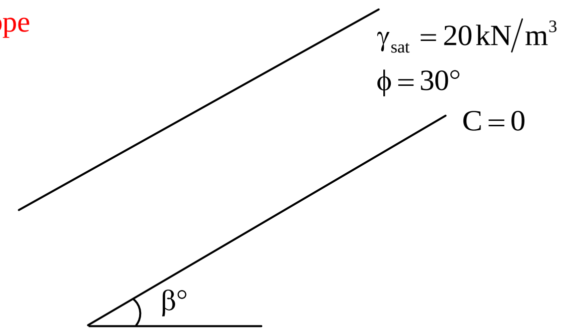
$$Q_u = Q_{pf}$$
$$= A_s.f_s = (\pi dL)\times(\propto C)$$
$$= \pi\times 0.6\times 12\times(0.5\times 110) = 1244.07kN$$

39. A granular soil has a saturated unit weight of 20 kN/m^3 and an effective angle of shearing resistance of 30°. The unit weight of water is 9.81 kN/m^3. A slope is to be made on this soil deposit in which the seepage occurs parallel to the slope up to the free surface. Under this seepage condition for a factor of safety of 1.5, the safe slope angle (in degree, round off to 1 decimal place) would be ______.

Key: **(11.09)**

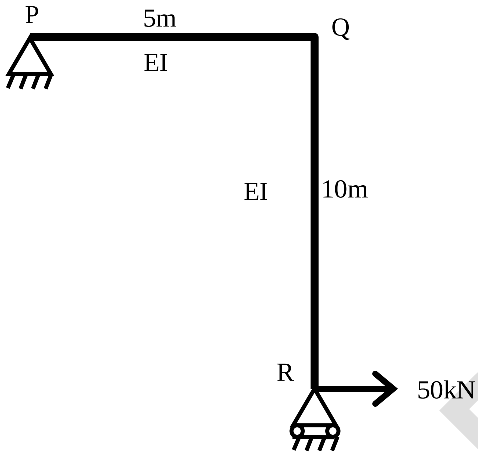
Sol: F.O.S = 1.5

If there is seepage parallel to the slope



$$F.O.S = \frac{C^1 + \gamma^1 z.\cos^2\beta.\tan\phi}{\gamma_{sat}.z.\cos\beta.\sin\beta}$$
$$F.O.S = \frac{\gamma^1 z.\cos^2\beta.\tan\phi}{\gamma_{sat}.z.\cos\beta.\sin\beta}$$
$$F.O.S = \frac{\gamma^1}{\gamma_{sat}}\frac{\tan\phi}{\tan\beta}$$
$$1.5 = \frac{(20-9.81)}{9.81}\frac{\tan 30}{\tan\beta}$$
$$\beta = 11.09°$$

40. A portal frame shown in figure (not drawn to scale) has a hinge support at joint P and a roller support at joint R. A point load of 50 kN is acting at joint R in the horizontal direction. The flexural rigidity. EI, of each member is 10^6 kNm^2. Under the applied load, the horizontal displacement (in mm, round off to 1 decimal place) of joint R would be _____



Key: (25)

Sol: $\Sigma F_x = 0$

$\Rightarrow H_P = F = 0 \Rightarrow H_P = -F$

$\Sigma M_P = 0$

$\Rightarrow V_R \times 5 + F \times 10 = 0 \Rightarrow V_R = -2F$

$\Sigma F_y = 0$

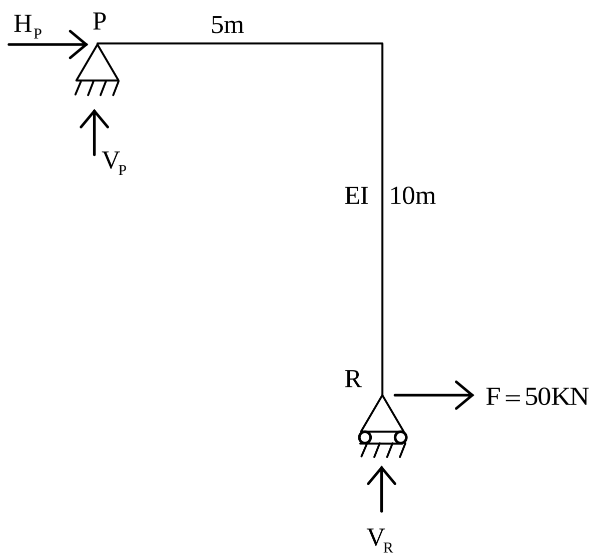
$\Rightarrow V_p + V_R = 0 \Rightarrow V_p = 2F$

$$\Delta R\big]\text{horizontal} = \frac{\partial U}{\partial F} = \frac{VU_{PQ}}{\partial F} + \frac{\partial V_{QR}}{\partial F}(\text{Castigliano's theorem})$$

$$U_{PQ} = \int \frac{M^2PQ}{2EI}dx;\ M_{PQ} = V_P x = 2F_x;\ x \in [0,5]$$

$$\Rightarrow \frac{\partial U_{PQ}}{\partial F} = \frac{1}{EI}\int \frac{\partial M_{PQ}}{\partial F} \times M_{PQ} dx = \frac{1}{EI}\int_0^5 2Fx \times 2x\ dx$$

$$= \frac{4F}{EI}\left[\frac{x^3}{3}\right]_0^5 = \frac{4 \times 50}{10^6} \times \frac{125}{3} = \frac{25}{3000}$$



$$U_{QR} = \int \frac{M_{QR}^2}{2EI}dx;\ M_{QR} = F_x;\ x \in [0,10]$$

$$\Rightarrow \frac{\partial U_{QR}}{\partial F} = \frac{1}{EI}\int \frac{\partial M_{QR}}{\partial F} M_{QR}\ dx = \frac{1}{EI}\int_0^{10} Fx \times x dx$$

$$= \frac{F}{EI}\left[\frac{x^3}{3}\right]_0^{10} = \frac{50}{10^6} \times \frac{1000}{3} = \frac{50}{3000}$$

$$\Rightarrow \frac{\partial U}{\partial F} = \frac{25}{3000} + \frac{50}{3000} = \frac{25}{1000} = 0.025\text{ m} = 25\text{mm}$$

$$\therefore \Delta R\big]\text{Horizontal} = \frac{\partial U}{\partial F} = 25\text{ mm}$$

41. A 0.80 m deep bed of sand filter (length 4m and width 3m) is made of uniform particles (diameter = 0.40 mm, specific gravity = 2.65, shape factor = 0.85) with bed porosity of 0.4. the bed has to be backwashed at a flow rate of 3.60 m^3/min. During backwashing, if the terminal

settling velocity of sand particles is 0.05 m/s, the expanded bed depth (in m, round off to 2 decimal places) is ______

Key: (1.21)

Sol: Expanded bed depth $= De = D\left[\dfrac{1-n}{1-n_e}\right]$

D=Depth of bed = 0.8m; n=Bed porosity = 0.4

$n_e =$ porosity of expanded bed $= \left(\dfrac{V_b}{V_s}\right)^{0.22}$

$V_s =$ terminal velocity of settling particles during back wasting $= 0.05\,m/s$

$V_b =$ backwash velocity

$= \dfrac{Q}{BL}(Q = \text{Back wash flow rate})$

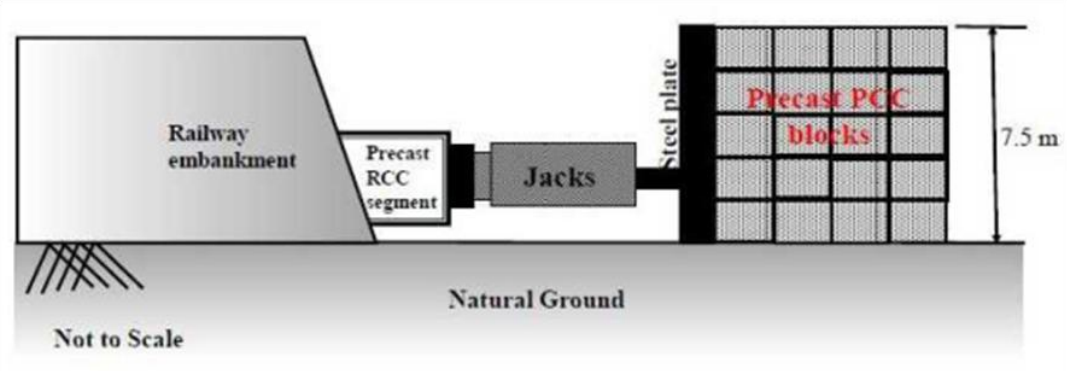
$\left[Q = 3.6\,m^3/\text{min}\ B = 3m\ L = 4m\right]$

$\Rightarrow V_b = \dfrac{3.6}{60\times 3\times 4} = 0.005\,m/s$

$\Rightarrow n_e = \left(\dfrac{0.005}{0.05}\right)^{0.22} = 0.60255$

$\therefore De = 0.8\left[\dfrac{1-0.4}{1-0.60255}\right] = 1.21\,m/s$

42. A 3 m × 3 m square precast reinforced concrete segments to be installed by pushing them through an existing railway embankment for making an underpass as shown in the figure. A reaction arrangement using precast PCC blocks placed on the ground is to be made for the jacks.



At each stage, the jacks are required to apply a force of 1875 kN to push the segment. The jacks will react against the rigid steel plate placed against the reaction arrangement. The footprint area of reaction arrangement on natural ground is 37.5 m^2. The unit weight of PCC block is 24 kN/m^3. The properties of the natural ground are: $c = 17\,kPa\ \phi = 25°$ and $\gamma = 18\,kN/m^3$.

Assuming that the reaction arrangement has rough interface and has the same properties that of soil, the factor of safety (round off to 1 decimal place) against shear failure is ______.

Key: (2.018)

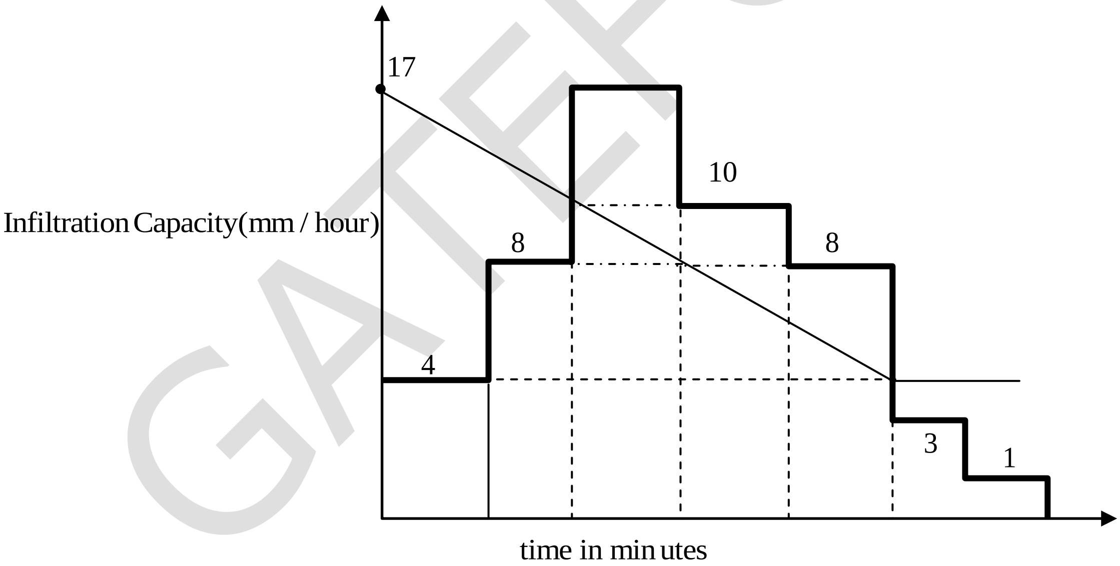
Sol: FOS against shear failure $= \dfrac{\text{strength}}{\text{Applied load}} = \dfrac{(c + \sigma \tan\phi)A}{P}$

$$\sigma = \frac{N}{A} = \frac{24\dfrac{kN}{m^3} \times 37.5m^2 \times 7.5m}{37.5m^2} = 24 \times 7.5\ kN/m^2$$

$$\Rightarrow FOS = \frac{(c + \sigma \tan\phi)A}{P} = \frac{(17 + 24 \times 7.5 \times \tan 25°) \times 37.5}{1875}$$

$$FOS = 2.0187$$

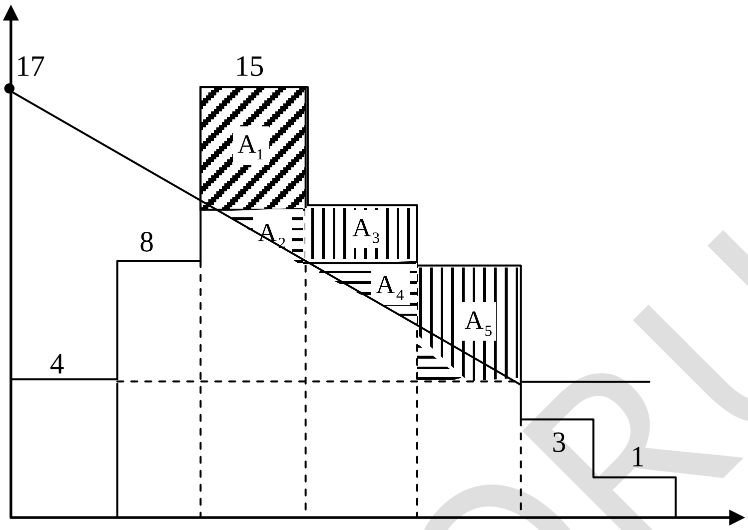
43. The hyetograph of a storm event of duration 140 minutes is shown in the figure.



The infiltration capacity at the start of this event (t = 0) is 17mm/hour, which linearly decreases to 10 mm/hour after 40 minutes duration. As the event progresses, the infiltration rate further drops down linearly to attain a value of 4mm/hour at t = 100 minutes and remains constant thereafter till the end of the storm event. The value of the infiltration index, ϕ (in mm/hour, round off to 2 decimal places), is _______.

Key: (7 .25)

Sol:



From the above diagram we can say that $A_6 = A_4$

$A_4 + A_5 = (A_5 + A_6)$

Infiltration capacity = 17 mm/hr

15mm/hr, 10mm/hr, and 8mm/hr rainfall will effect the run off the, shaded area will shows the effective rainfall i.e runoff

Total runoff

$$= A_1 + A_2 + A_3 + A_4 + A_5$$
$$= A_1 + A_2 + A_3 + (A_5 + A_6)$$
$$= \left(\frac{5}{60} \times 20\right) + \frac{1}{2}\left(\frac{2}{60} \times 20\right) + \left(\frac{2}{60} \times 20\right) + \left(\frac{4}{60} \times 20\right)$$
$$= 4\text{mm}$$

Precipitation $= (15+10+8)\text{mm/hr}\ \frac{20}{60} = 11\text{mm}$

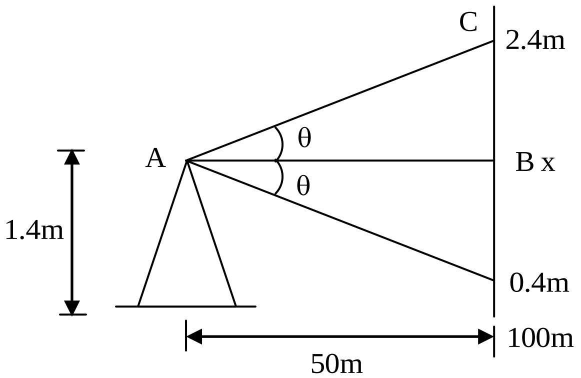
Now assuming $4 < \phi < 8$

$$(8-\phi)\frac{20}{60} + (15-\phi)\frac{20}{60} + (10-\phi)\frac{20}{60} + (8-\phi)\frac{20}{60} = 4$$
$$\phi = 7.25\text{mm / hr}$$

44. A staff is placed on a benchmark (BM) of reduced level (RL) 100.000 m and a theodolite is placed at a horizontal distance of 50m from the BM to measure the vertical angles. The measured vertical angles from the horizontal at the staff readings of 0.400m and 2.400 m are found to be the same. Taking the height of the instrument as 1.400 m, the RL (in m) of the theodolite station is _______.

Key: (100)

Sol:



$$\tan\theta = \frac{2.4 - x}{50} = \frac{x - 0.4}{50}$$

$$2x = 2.8$$

$$x = 1.4\text{m}$$

$$\text{H.O.I} = 100 + 1.4 = 101.4\text{m}$$

RL of theodelite station = 101.4 – theodelite hight = 101.4 – 1.4 = 100m

45. A rectangular open channel has a width of 5m and a bed slope of 0.001. For a uniform flow of depth 2m, the velocity is 2m/s. The Manning's roughness coefficient for the channel is

(A) 0.017 (B) 0.050 (C) 0.033 (D) 0.002

Key: (A)

Sol:

$$\text{width}(B) = 5\text{m}$$

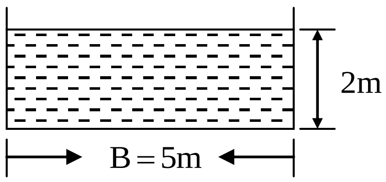
$$\text{depth}(y) = 2\text{m}$$

$$\text{Area}(B \times y) = 5 \times 2 = 10\text{m}^2$$

$$\text{Perimeter}(R) = B + 2y = 5 + 2 \times 2 = 9\text{m}$$

$$\text{Hydraulicrodius}(R) = \frac{\text{Area}}{\text{Perimeter}}$$

$$R = \frac{10}{9} = 1.11$$



By the manning's equation

$$\text{Velocity}(V) = \frac{1}{n} R^{2/3} S^{1/2}$$

$$2 = \frac{1}{n}(1.11)^{2/3}(0.001)^{1/2}$$

$$n = \frac{1.072 \times 0.03162}{2} = 0.0169 \cong 0.017$$

Manning's roughness coefficient (n) = 0.017

46. Consider a laminar flow in the x-direction between two infinite parallel plates (Couette flow). The lower plate is stationary and the upper plate is moving with a velocity of 1 cm/s in the x-

direction. The distance between the plates is 5mm and the dynamic viscosity of the fluid is 0.01 N-s/m^2. If the shear stress on the lower plate is zero, the pressure gradient, $\frac{\partial p}{\partial x}$, (in N/m^2 per m, round off to 1 decimal place) is_________.

Key: (8)

Sol: Given data;

Velocity of plate, $V = 1\,\text{cm/sec}$

Distance between the late = 5 mm

Dynamic viscosity of fluid $= 0.01\,\text{N} - \text{S/m}^2$

Shear stress at lower plate = 0

Pressure gradient $\frac{\partial P}{\partial x} = ?$

We Know that, in case of coquette flow, shear stress (τ) is given by

$$\tau = \frac{\mu V}{B} + \left(-\frac{\partial P}{\partial x}\right)\left(\frac{B}{2} - y\right)$$

At lower plate, $y = 0;\ \tau = 0\,[\text{Given}]$

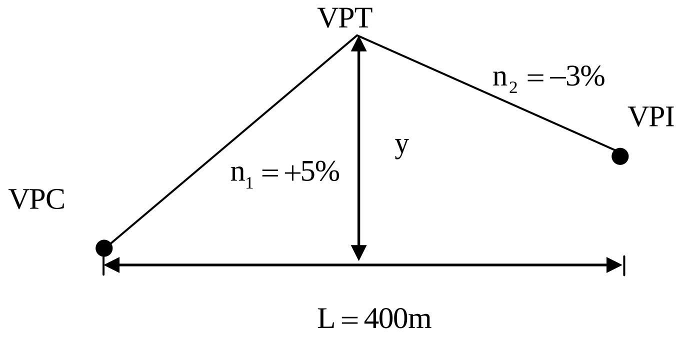
$$0 = \frac{0.01 \times 0.01}{0.005}\left(\frac{\partial P}{\partial x}\right)\left[\frac{0.005}{2} - 0\right]$$

$$\frac{\partial P}{\partial x} = 8\,\text{N/m}^2 \text{ per m}$$

47. A parabolic vertical curve is being designed to join a road of grade + 5% with a road of grade – 3%. The length of the vertical curve is 400 m measured along the horizontal. The vertical point of curvature (VPC) is located on the road of grade +5%. The difference in height between VPC and vertical point of intersection (VPI) (in m, round off to the nearest integer) is ____

Key: (10)

Exp:



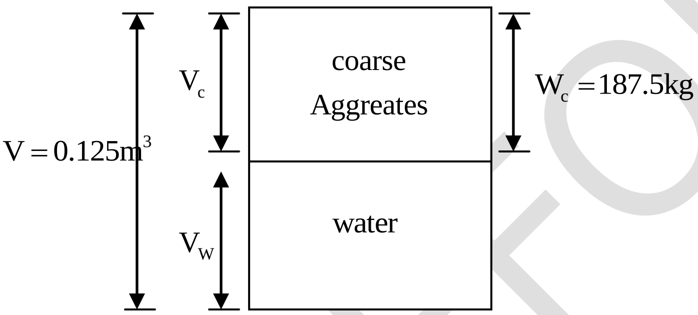
Vertical distance between VPC and VPT

$$y = \frac{L}{2}n_1 = \frac{400}{2} \times \frac{5}{100} = 10m$$

48. A box measuring 50 cm × 50 cm × 50 cm is filled to the top with dry coarse aggregate of mass 187.5 kg. The water absorption and specific gravity of the aggregate are 0.5% and 2.5, respectively. The maximum quantity of water (in kg, round off to 2 decimal places) required to fill the box completely is _____.

Key: (50.94)

Sol: Volume of box $= 0.5 \times 0.5 \times 0.5 = 0.125m^3$



$$\text{Volume}(V) = V_C + V_w$$

$$\text{Volume of coarse aggregates } (V_c) = \frac{W_c}{G.\gamma_w}$$

$$= \frac{187.5\text{ kg}}{2.5 \times 1000\text{ kg/m}^3} = 0.075\text{ m}^3$$

$$\text{Volume of water} = V - V_c = 0.125 - 0.075 = 0.05m^3$$

Water absorption is 0.5%

$$\text{Water absorbed } = 0.5 \times \frac{187.5}{100} = 0.9375\text{kg}$$

$$\text{Toatl weight of water} = \text{volume} \times \gamma_w + \text{weight of water absorbed}$$
$$= 0.05 \times 1000 + 0.9375 = 50.9375\text{ kg}$$

49. A sample of air analysed at 0°C and 1 atm pressure is reported to contain 0.02 ppm (parts per million) of NO_2. Assume the gram molecular mass of NO_2 as 46 and its volume at 0°C and 1 atm pressure as 22.4 litres per mole. The equivalent NO_2 concentration (in microgram per cubic meter, round off to 2 decimal palces) would be ________.

Key: (41.07)

Sol: $1\text{ ppm } NO_2 = \frac{1\text{ part of } NO_2}{10^6\text{ parts of air}} = \frac{1m^3\text{ of } NO_2}{10^6\text{ m of air}}$

1 mole of NO_2 occupies 22.4 litres of volume at STP.

$\Rightarrow$ 1 mole of NO_2 occupies 0.0224 m^3 volume at STP.

$\Rightarrow\ 1m^3$ of NO_2 contains $\dfrac{1}{0.0224}$ moles = 44.64 moles

1 mole of NO_2 = 46 gms

$\Rightarrow 1m^3 \text{ of } NO_2 = 44.64 \times 46 \text{ gms} = 2053.57 \text{ gms}$

$\Rightarrow 0.02m^3 \text{of } NO_2 = 41.07 \text{ gms of } NO_2$

$$\therefore \quad 0.02 \text{ ppm of } NO_2 = \frac{0.02m^3 \text{ of } NO_2}{10^6 m^3 \text{of air}} = 41.07 \text{gms per } 10^6 m^3$$

$$= 41.07 \times 10^{-6}\ gm/m^3 = 41.07\ \mu g/m^e$$

50. A survey line was measured to be 285.5m with a tape having a nominal length of 30m. On checking, the true length of the tape was found to be 0.05 m too short. If the line lay on a slope of 1 in 10, the reduced length (horizontal length) of the line for plotting of survey work would be

(A) 285.6m (B) 283.6m (C) 285.0m (D) 284.5m

Key: (B)

Sol: Measured distance $(\ell) = 285.5m$

Length of tape = 30m

True length of tape = 30m -0.05=29.95

Correct length $(\ell^1) = \dfrac{29.95}{300} \times 28.5.5 = 285.024\, n$

The line measured on 1/10 slope

Correction due to slope

$$= \frac{h^2}{2\ell} = \frac{\left(\dfrac{285.024}{10}\right)^2}{2 \times 285.024} = 1.425$$

corrected length $= 285.024 - 1.425 = 283.598m$

51. A one-dimensional domain is discretized into N sub-domains of width Δx with node numbers i = 0, 1, 2, 3,....,N. If the time scale is discretized in steps of Δt, the forward-time and centered-space finite difference approximation at i^{th} node and n^{th} time step, for the partial differential equation $\dfrac{\partial v}{\partial t} = \beta \dfrac{\partial^2 v}{\partial x^2}$ is

(A) $\dfrac{v_{i+1}^{(n+1)} - v_i^{(n)}}{\Delta t} = \beta\left[\dfrac{v_{i+1}^{(n)} - 2v_i^{(n)} + v_{i-1}^{(n)}}{2\Delta x}\right]$

(B) $\dfrac{v_i^{(n)} - v_i^{(n-1)}}{2\Delta t} = \beta\left[\dfrac{v_{i+1}^{(n)} - 2v_i^{(n)} + v_{i-1}^{(n)}}{2\Delta x}\right]$

(C) $\frac{v_i^{(n+1)} - v_i^{(n)}}{\Delta t} = \beta\left[\frac{v_{i+1}^{(n)} - 2v_i^{(n)} + v_{i-1}^{(n)}}{(\Delta x)^2}\right]$ (D) $\frac{v_i^{(n)} - v_i^{(n-1)}}{\Delta t} = \beta\left[\frac{v_{i+1}^{(n)} - 2v_i^{(n)} + v_{i-1}^{(n)}}{(\Delta x)^2}\right]$

Key: (A)

Sol: $\frac{\partial v}{\partial t} = \beta\frac{\partial^2 V}{\partial x^2}$

$\frac{\partial v}{\partial t} = \frac{V_i^{(n+1)} V_i^{(n)}}{\Delta t}$ (using forward time finite difference approximation)

Also; $f^{11}(x) = \frac{\partial^2 f}{dx^2} = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$

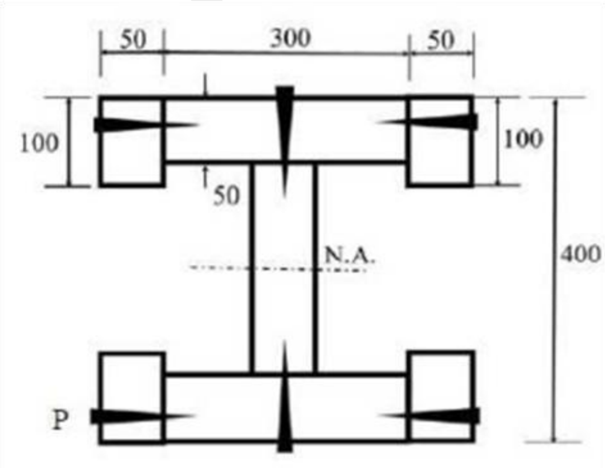
(Using centered space finite difference approximation)

$\Rightarrow \frac{\partial^2 v}{\partial x^2} = \frac{V_{i+1}^{(n)} - 2V_i^{(n)} + V_{i-1}^{(n)}}{(\Delta x)^2}$

$\therefore \frac{\partial v}{\partial t} = \beta\frac{\partial^2 v}{\partial x^2}$ can be represented as

$\frac{V_i^{(n+1)} - V_i^{(n)}}{\Delta t} = \beta\left[\frac{V_{i+1}^{(n)} - 2V_i^{(n)} + V_{i-1}^{(n)}}{(\Delta x)^2}\right]$

52.



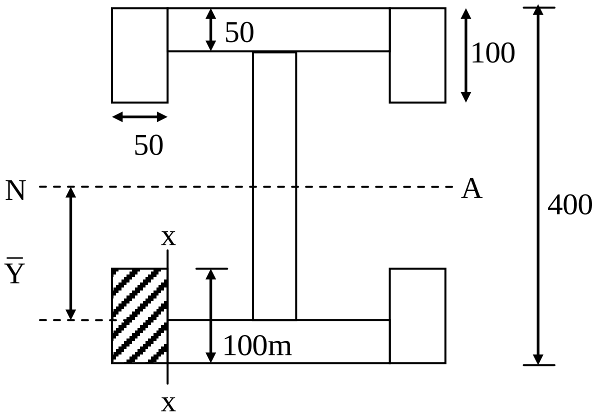
All dimensions are in mm

The cross-section of a built-up wooden beam as shown in the figure (not drawn to scale) is subjected to a vertical shear force of 8kN. The beam is symmetrical about the neutral axis (N.A.) shown, and the moment of inertia about N.A. is $1.5 \times 10^9 mm^4$. Considering that the nails at the location P are spaced longitudinally (along the length of the beam) at 60 mm, each of the nails at P will be subjected to the shear force of

(A) 240 N (B) 480 N (C) 120 N (D) 60 N

Key: (A)

Sol:



Shear flow at $x-x \quad = \dfrac{SA\bar{y}}{I}$

$S = \text{shear forece} = 8kN = 8000\ N$

A = Area of shaded portion $= 50 \times 100\ mm^2$

$\bar{Y} =$ Distance of centroid of shaded portion form N.A

$= 200 - \dfrac{100}{2} = 150\ mm$

$I = 1.5 \times 10^9\ mm^4$

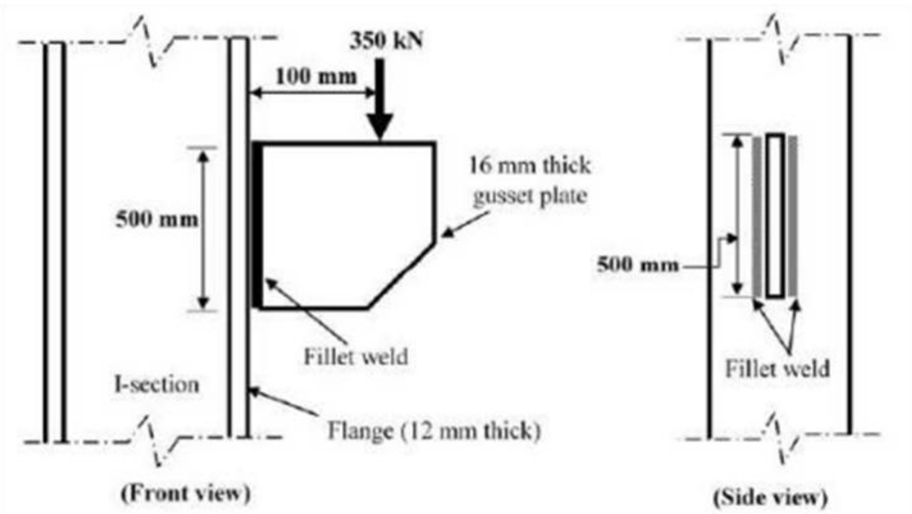
$\Rightarrow$ shear flow at $x - x = \dfrac{8000 \times 50 \times 1000 \times 150}{1.5 \times 10^9} = 40\ N/mm$

This shear flow acts longitudinally throughout section x –x.

$\Rightarrow$ Shear resisted by nails at P = shear flow x sparing of nails

$= 40 \times 60 = 240N$

53. A 16 mm thick gusset plate is connected to the 12mm thick flange plate of an I-section using fillet welds on both sides as shwon in the figure (not drawn to scale). The gusset plate is subjected to a point load of 350 kN acting at a distance of 100 mm from the flange plate. Size of fillet weld is 10 mm.

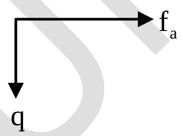


The maximum resultant stress (in MPa, round off to 1 decimal place) on the fillet weld along the vertical plane would be ________.

Key: (105.35)

Sol: As per IS 800: 2007

Equivalent maximum resultant stress due to combination of normal & shear stress



$$= f_e = \sqrt{f_a^{\,2} + 3q^2}$$

f_a = normal stress due to bending

q = shear stress

$$q = \frac{\text{shear force}}{\text{shear area}} = \frac{350\text{ kN}}{dt + dt}$$

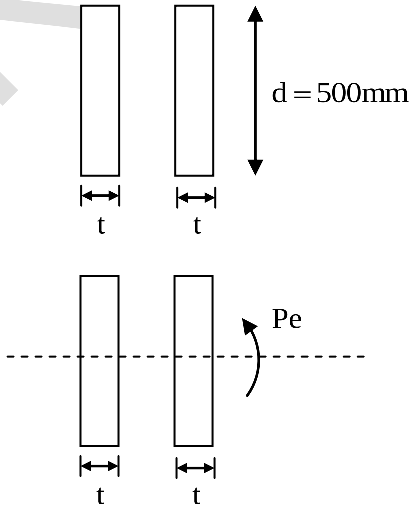
t = effective throat thickness = ks (s = size of weld)

$$\Rightarrow t = 0.7 \times 10 = 7\text{mm}$$

$$\Rightarrow q = \frac{350 \times 1000}{2 \times 10 \times 500} = 50\text{ N/mm}^2$$

$$f_a = \frac{Pe}{z} = \frac{350 \times 1000 \times 100}{2 \times 7 \times \frac{(500)^2}{6}} = 60\text{ N/mm}^2 \qquad \left(Z = \frac{(2t)d^2}{6}\right)$$

$$\Rightarrow f_e = \sqrt{60^2 + 3 \times 50^2} = 105.35\text{ MPa}$$



54. Consider two functions: $x = \psi \ell n \phi$ and $y = \phi \ell n \psi$. Which one of the following is the correct expression for $\frac{\partial \psi}{\partial x}$?

(A) $\dfrac{\ell n \psi}{\ell n \phi \ell n \psi - 1}$ (B) $\dfrac{\ell n \phi}{\ell n \phi \psi - 1}$ (C) $\dfrac{x \ell n \psi}{\ell n \phi \psi - 1}$ (D) $\dfrac{x \ell n \phi}{\ell n \phi \ell n \psi - 1}$

Key: (A)

Sol: Given, $x = \psi \ell n \phi$ & $y = \phi \ell n \psi$

$$\Rightarrow \psi = \frac{x}{\ell n \phi};$$

$$\Rightarrow \psi = \frac{x}{\ell n \left[\frac{y}{\ell n \psi}\right]} \qquad \left(\begin{array}{l} \because y = \phi \ell n\, \psi \\ \Rightarrow \phi = \frac{y}{\ell n \psi} \end{array}\right]$$

$$\Rightarrow \psi = \frac{x}{\ell n[y] - \ell n[\ell n \psi]} \qquad \left(\because \ell n\left[\frac{A}{B}\right] = \ell n(A) - \ell n(B)\right]$$

$\therefore$ Differentiating ψ partially w.r.t ‘x’, we have

$$\frac{\partial\psi}{\partial x}=\frac{\left[\left(\ell n(y)-\ell n[\ell n\,\psi]\right)(1)-x\left[0-\frac{1}{\ell n\psi}\frac{1}{\psi}\frac{\partial\psi}{\partial x}\right]\right]}{\left[\ell n(y)-\ell n(\ell n\psi)\right]^2};\ \text{since y is constant}$$

$$=\frac{\frac{x}{\psi}+x\left[\frac{1}{\ell n\psi}\frac{1}{\psi}\frac{\partial\psi}{\partial x}\right]}{\left(\frac{x}{\psi}\right)2};\ \text{since } \ell ny-\ell n(\ell n\psi)=\ell n\phi=\left[\frac{x}{\psi}\right]$$

$$\Rightarrow\frac{\partial\psi}{\partial x}=\frac{1+\frac{1}{\ell n\psi}\frac{\partial\psi}{\partial x}}{\frac{x}{\psi}}\Rightarrow\frac{x}{\psi}\frac{\partial\psi}{\partial x}=1+\frac{1}{\ell n\psi}\frac{\partial\psi}{\partial x}$$

$$\Rightarrow\frac{x}{\psi}\frac{\partial\psi}{\partial x}-\frac{1}{\ell n\psi}\frac{\partial\psi}{\partial x}=1$$

$$\Rightarrow\frac{\partial\psi}{\partial x}\left[\frac{x}{\psi}-\frac{1}{\ell n\psi}\right]=1\Rightarrow\frac{\partial\psi}{\partial x}\left[\frac{x\ell n\psi-\psi}{\psi\ell n\psi}\right]=1$$

$$\Rightarrow\frac{\partial\psi}{\partial x}=\frac{\psi\ell n\psi}{x\ell n\psi-\psi}=\frac{\ell n\psi}{\frac{x}{\psi}\ell n\psi-1}=\frac{\ell n\psi}{\ell n\phi\ell n\psi-1};$$

$$\text{Since }\frac{x}{\psi}=\ell n\,\phi\Rightarrow\frac{\partial\psi}{\partial x}=\frac{\ell n\psi}{\ell n\phi\ell n\psi-1}$$

55. Tie bars of 12 mm diameter are to be provided in a concrete pavement slab. The working tensile stress of the tie bars is 230 MPa, the average bond strength between a tie bar and concrete is 2 MPa, and the joint gap between the slab is 10mm. Ignoring the loss of bond and the tolerance factor, the design length of the tie bars (in mm, round off to the nearest integer) is ______.

Key: (700)

Sol: Length of Tie bars

$=2L_d+\text{joint gap}$

$=2L_d+10\qquad\left[L_d=\frac{\phi\sigma_{st}}{4\tau_{bd}}\right]$

$=\frac{2\phi\sigma_{st}}{4\tau_{bd}}+10=\frac{2\times12\times230}{4\times2}+10$

$=700\text{ mm}$